HIGH PERFORMANCE PARALLEL ALGORITHMS FOR IMPROVED REDUCED-ORDER MODELING

AFOSR FA9550-05-1-0449

FINAL REPORT

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OBJECTIVES

The primary objectives of this research is to develop reliable parallel algorithms and software tools for building improved reduced-order models with an emphasis on fluid systems.

ACCOMPLISHMENTS

Overview

Our research program has focused on reduced-order models for large-scale systems with an emphasis on fluids and control problems. A major accomplishment has been to develop and analyze an algorithm suitable for computing proper orthogonal decomposition (POD) basis functions from large-scale CFD data. The main feature of this algorithm is that it can be used with highly scalable CFD algorithms that utilize domain decomposition to distribute data across compute nodes. In other words, it works with data sets, allowing POD to be used with complex 3D flows. This algorithm was tested by computing reduced-order models for unsteady flow past a 3D cylinder computed on System X (Virginia Tech's high performance supercomputer which was ranked number 3 in the world in November 2003 and was still in the top 50 in 2006). This computation would not have been possible if the data had to be collected on one compute node.

We have also investigated a number of traditional methods for reducing the computational model in complex flows. These include a multilevel approach that can be used to split the computation between coarse and fine grids. An added feature of this multilevel approach is that it gave the first theoretical insight into the relationship between the filter radius in an LES model and the required discretization size. Traditionally, using the relationship of two discretization points in each direction inside a filter radius had been used based on computational experience. We have also studied accuracy issues in computing sensitivity analysis for fluids. This research will be applied in the follow on proposed research incorporating sensitivity analysis with POD in order to extend the accuracy of reduced order models in parameter space.

For large linear systems, alternative projection-based methods based on a rational Krylov projection framework were proposed and studied. For control problems, or model reduction of linear input-output systems, these projection methods have optimal $\mathcal{H}_2$ properties. This projection framework has also been applied to the Fourier model reduction methodology

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14. ABSTRACT
We describe the development of a reliable parallel algorithm and software tools that utilize flow-adapted KLE/POD representations and that are able to take advantage of distributed data formats on cluster/grid computer architectures. The associated module functions efficiently within the context of current best practices of fluid flow simulation. Additionally, we describe methods that lead to greater predictive capability and extend the range of flows for which KLE/POD-based methods are effective. Model reduction methods used for linear input-output systems based on a rational Krylov framework were also studied. We propose new investigations informed both by approximate inertial manifold approaches and by energy-based turbulence modeling/LES approaches capable of accounting for the effect of small scale dynamic structures.

15. SUBJECT TERMS
Model reduction, Karhunen-Loeve expansion, proper orthogonal decomposition, parallel algorithm, computational fluid dynamics, control

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as well. Toward the end of this research study, we investigated an interpolatory framework that provides insight on the optimal parameter values in which to sample data.

We will provide detail on the high-performance computational algorithm for parallel computation of POD basis vectors as well as results from its application to 3D flow past a cylinder at a Reynolds number of 525.

The Filtered Subspace Iteration Algorithm

An enabling method for simulating complex PDE dynamical systems is *domain decomposition* (see Widlund and Tosseli for a good overview). The general philosophy is to partition the spatial domain into subdomains and integrate the PDE in parallel on each subdomain. "Information" at the boundaries of these subdomains are communicated to their neighbors to facilitate the integration. There are many approaches for implementing domain decomposition that we will not discuss here. In practice, these subdomains are constructed so that the size of the discrete systems are nearly the same (for good load balancing) and minimize the discretization size along the interfaces (for low communication requirements).

At the end of a simulation, the snapshot matrix is distributed across several processors. We present an algorithm for computing the SVD that takes advantage of this data structure. This is of interest since it may not be possible to assemble the entire snapshot matrix on a single processor.

We assume that the POD snapshot matrix can be represented as

$$\mathcal{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_{n_p} \end{bmatrix} \qquad \begin{array}{ccc} Y_1 & \text{is} & N_1 \times p \\ Y_2 & \text{is} & N_2 \times p \\ & \vdots & \\ Y_{n_p} & \text{is} & N_{n_p} \times p \end{array}$$

The algorithm provides an efficient method for computing the dominant q POD basis vectors where we assume

$$q << p << N_i << N = \sum_{i=1}^{n_p} N_i.$$

we use caligraphic notation $(\mathcal{Y}, \mathcal{U}, \mathcal{V})$ to denote global quantities, subscripts to denote processor number Y_1, Y_2, $\ldots$, superscripts to denote iteration number and $\hat{}$ to denote an aggregated matrix. The algorithm, known as *filtered subspace iteration* consists of two parts. The first constructs a good initial guess and the second performs iterative corrections to it.

Algorithm 1 *Initialization*

for $i = 1 : n_p$ $V_i^{(0)}$ = q *dominant* right *singular vectors of* Y_i *send* $V_i^{(0)}$ *to other processors (or to processor 0) end*

Accumulate $\hat{\mathcal{V}} = \left[V_1^{(0)} V_2^{(0)} \cdots V_{n_p}^{(0)}\right]$

$\mathcal{V}^{(0)}$ = q *dominant* left *singular vectors of* $\hat{\mathcal{V}}$.

broadcast $\mathcal{V}^{(0)}$ *to all processors (if not locally computed)*

Compute $\mathcal{C}^{(0)} = \mathcal{Y}\mathcal{V}^{(0)}$ *(in parallel)*

Compute $\mathcal{U}^{(0)}$ = q *dominant* left *singular vectors of* $\mathcal{C}^{(0)}$ *using Algorithm 2 or 3 below.*

This algorithm relies on the fact that the snapshot matrix comes from a dynamical system and that solutions would have the "separation of variables" structure

$$y(t) = \sum_{i=1}^{\infty} y_i a_i(t), \qquad t \in (t_a, t_b).$$

In other words, we assume that the dominant time trajectory $a_1(\cdot)$ is prominent for each portion of y_1. Numerical experiments indicate that there is a strong correlation in the right singular vectors of Y_i. Thus, this initialization provides a good starting point for the *filtered subspace iteration* algorithm (Algorithm 4).

One implementation note is that the accumulation of $\hat{\mathcal{V}}$ could occur on every processor (through broadcasts of the $V_i^{(0)}$'s). Hence, each processor would find $\mathcal{V}^{(0)}$ and avoid the subsequent broadcast. This avoids two communication steps at the expense of one more network intensive step. This modification has not yet been tested. A second note is that the $\mathcal{U}^{(0)}$ only needs to be an orthonormal basis for $\mathcal{C}^{(0)}$ in practice, thus could be found by Gram-Schmidt or something equivalent. However, using an SVD at this steps gives good agreement with the true POD basis and hence is useful in comparisons.

We now present two approaches for computing $\mathcal{U}^{(j)}$, the q dominant singular vectors of $\mathcal{C}^{(j)}$ (an $N \times q$ matrix).

Algorithm 2 *Real Schur*

1. *Compute* $\left(\mathcal{C}^{(j)}\right)^T \mathcal{C}^{(j)}$. *requires one time communication of approximately* $N \times q$.
2. *Perform a real Schur factorization of this* $q \times q$ *matrix product*

$$\left(\mathcal{C}^{(j)}\right)^T \mathcal{C}^{(j)} = Z S^2 Z^T.$$

3. *Broadcast* Z *and the diagonal elements of* S *to all processors.*
4. *The multiplication* $\mathcal{Y} Z S^{-1}$ *places the correct rows of* $\mathcal{U}^{(j)}$ *onto the appropriate processor.*

Note that the product $\mathcal{C}^T\mathcal{C}$ above can be performed in parallel with relatively little communication. If $\mathcal{C}$ has the following structure

$$\mathcal{C}^T = \left[C_1^T \; C_2^T \; \cdots \; C_{n_p}^T \right],$$

then $\mathcal{C}^T\mathcal{C}$ can be expressed as the sum

$$C_1^T C_1 + C_2^T C_2 + \cdots + C_{n_p}^T C_{n_p}$$

where the ith term above can be computed locally on the ith processor. Thus, there only needs to be parallel communication of the n_p distinct $q \times q$ matrices.

Algorithm 3 *Gram-Schmidt Orthogonalize the columns of* $\mathcal{C}^{(j)}$.

The result of the initialization algorithm is a good approximation to the dominant q dimensional subspace for the range of $\mathcal{Y}$. In practice, this provides a great start for the iterative corrections given in the filtered subspace iteration algorithm below.

Algorithm 4 (Filtered Subspace Iteration) *for j=1:Jmax*

1. *Calculate* $\mathcal{D}^{(j)} = \left(\mathcal{U}^{(j-1)}\right)^T \mathcal{Y}$. *This* $q \times p$ *matrix can be accumulated onto one processor or broadcast to every processor.*

2. $\mathcal{V}^{(j)}$ = q *dominant* right *singular vectors of* $\mathcal{D}^{(j)}$. *Possibly communicate to every processor.*

3. *Calculate* $\mathcal{C}^{(j)} = \mathcal{Y}\mathcal{V}^{(j)}$ *(an* $N \times q$ *matrix).*

4. *Compute* $\mathcal{U}^{(j)}$ *as the* q *dominant* left *singular vectors of* $\mathcal{C}^{(j)}$ *using one of the two algorithms above.*

Test for convergence.

Analysis of the filtered subspace iteration

At the end of the first step in Algorithm 4, we have

$$\begin{aligned} \mathcal{D}^{(j)} &= \left(\mathcal{U}^{(j-1)}\right)^T \mathcal{Y} \\ &= \hat{\mathcal{U}}^{(j)}\hat{\Sigma}^{(j)} \left[\mathcal{V}^{(j)}\tilde{\mathcal{V}}^{(j)}\right]^T \end{aligned}$$

(since $\mathcal{V}^{(j)}$ are the dominant right singular vectors of $\mathcal{D}^{(j)}$).

Rearranging terms, we have

$$\left(\mathcal{U}^{(j-1)}\right)^T \mathcal{Y} \left[\mathcal{V}^{(j)}\tilde{\mathcal{V}}^{(j)}\right] = \hat{\mathcal{U}}^{(j)}\hat{\Sigma}^{(j)}$$

By filling out the orthogonal complement to $\mathcal{U}^{(j-1)}$, we have

$$\begin{bmatrix} \left(\mathcal{U}^{(j-1)}\right)^T \\ \left(\tilde{\mathcal{U}}^{(j-1)}\right)^T \end{bmatrix} \mathcal{Y} \left[\mathcal{V}^{(j)}\tilde{\mathcal{V}}^{(j)}\right] = \begin{bmatrix} \left(\mathcal{U}^{(j-1)}\right)^T \mathcal{Y}\mathcal{V}^{(j)} & \color{red}{\left(\mathcal{U}^{(j-1)}\right)^T \mathcal{Y}\tilde{\mathcal{V}}^{(j)}} \\ \left(\tilde{\mathcal{U}}^{(j-1)}\right)^T \mathcal{Y}\mathcal{V}^{(j)} & \left(\tilde{\mathcal{U}}^{(j-1)}\right)^T \mathcal{Y}\tilde{\mathcal{V}}^{(j)} \end{bmatrix}$$

Note that the (1,2) block above (in red) is a zero block since $\mathcal{U}^T\mathcal{Y}$ is a rank q matrix and the columns $q+1 \rightarrow p$ of $\hat{\mathcal{U}}^{(j)}\hat{\Sigma}^{(j)}$ are zero.

Thus, we observe that

$$\begin{bmatrix} \left(\mathcal{U}^{(j-1)}\right)^T \\ \left(\tilde{\mathcal{U}}^{(j-1)}\right)^T \end{bmatrix} \mathcal{Y}\mathcal{Y}^T \left[\left(\mathcal{U}^{(j-1)}\right)^T \left(\tilde{\mathcal{U}}^{(j-1)}\right)^T\right] = \begin{bmatrix} Y_{11}^{(j-1)} & 0 \\ Y_{12}^{(j-1)} & Y_{22}^{(j-1)} \end{bmatrix} \begin{bmatrix} \left(Y_{11}^{(j-1)}\right)^T & \left(Y_{12}^{(j-1)}\right)^T \\ 0 & \left(Y_{22}^{(j-1)}\right)^T \end{bmatrix}.$$

Likewise, after the second step of the filtered subspace algorithm, we have

$$\mathcal{C}^{(j)} = \mathcal{Y}\mathcal{V}^{(j)} = \left[\mathcal{U}^{(j)}\tilde{\mathcal{U}}^{(j)}\right] \tilde{\Sigma}\tilde{V}^T.$$

and

$$\begin{bmatrix} \left(\mathcal{U}^{(j)}\right)^T \\ \left(\tilde{\mathcal{U}}^{(j)}\right)^T \end{bmatrix} \mathcal{Y} \left[\mathcal{V}^{(j)}\tilde{\mathcal{V}}^{(j)}\right] = \begin{bmatrix} \left(\mathcal{U}^{(j)}\right)^T \mathcal{Y}\mathcal{V}^{(j)} & \left(\mathcal{U}^{(j)}\right)^T \mathcal{Y}\tilde{\mathcal{V}}^{(j)} \\ \color{red}{\left(\tilde{\mathcal{U}}^{(j)}\right)^T \mathcal{Y}\mathcal{V}^{(j)}} & \left(\tilde{\mathcal{U}}^{(j)}\right)^T \mathcal{Y}\tilde{\mathcal{V}}^{(j)} \end{bmatrix}$$

Again, the (1,2) block above (in red) is a zero block since $\tilde{\Sigma}^{(j)}\left(\tilde{\mathcal{V}}^{(j)}\right)^T$ is a rank q matrix and columns $q+1 \rightarrow p$ are zero.

Application of the filtered subspace iteration

This algorithm was tested on the three dimensional flow past a circular cylinder with a fine mesh in the streamwise and crossflow plane $(128 \times 192 \times 32)$ leading to 3.14×10^6 degrees of freedom partitioned over 128 processors. Data was generated over 80 time snapshots and the above algorithm was applied. The first two POD modes for the velocity are depicted in Figures 1, 2 and 3. We note that a study of the accuracy of the algorithm was performed in the paper *A Domain Decomposition Approach to POD* by the co-PIs.

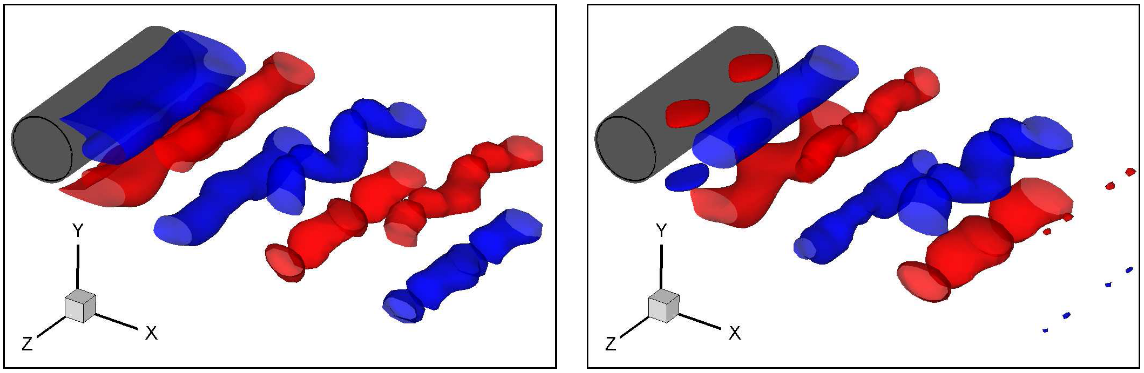

Figure 1: Streamwise POD Modes 1 and 2

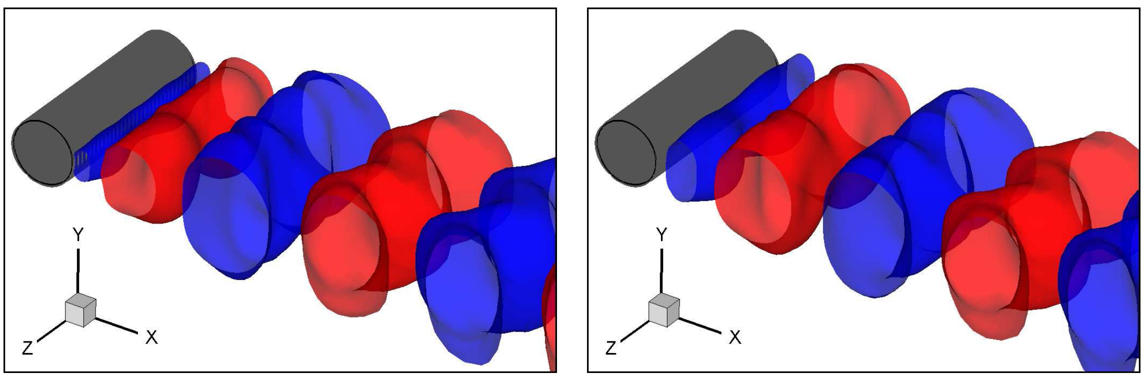

Figure 2: Crossflow POD Modes 1 and 2

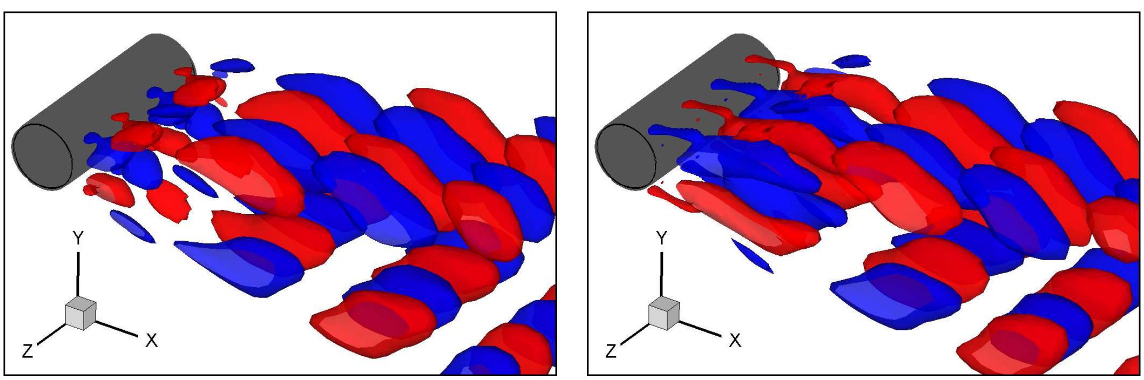

Figure 3: Spanwise POD Modes 1 and 2

Continuing Research

Two of the co-PIs, Jeff Borggaard and Traian Iliescu along with two postdoctoral associates (Andrew Duggleby in Mechanical Engineering and Alexander Hay in Mathematics) are using turbulent flow data to generate a reduced-order basis for improved reduced-order models for turbulent flows. We are currently validating the reduced-order models we proposed in challenging three-dimensional simulations of turbulent pipe flows. We have also proposed alternative approaches to the usual reduced-order modeling approaches, by using the Variational Multiscale method and the Dynamic procedure (based on a two-scale computation of the turbulent model parameters).

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Project Summary

Personnel Supported

Faculty
Chris Beattie, Jeff Borggaard, Serkan Gugercin and Traian Iliescu (co-PIs)

Post-Docs
Andrew Duggelby, Alexander Hay and Sonja Schlaugh

Students
Weston Hunter, Denis Kovacs, Miroslav Stoyanov and Daniel Sutton

Dissemination of Research Results

Publications

During this project, we have submitted more than 32 publications. The publication topics include model reduction, control, fluid dynamics and sensitivity analysis.

1. C.A. Beattie, J. Borggaard, S. Gugercin, and T. Iliescu, A domain decomposition approach to POD, *Proceedings of 45th IEEE CDC*, pp. 6750–6756 (2006).

2. C. Beattie and S. Gugercin, Krylov-based model reduction of second-order systems with proportional damping, *Proceedings of the 44th IEEE CDC*, (2005).

3. C.A. Beattie and S. Gugercin, Inexact solves in Krylov-based model reduction, *Proceedings of the 45th IEEE Conference on Decision and Control*, pp. 3405–3411 (2006).

4. C.A. Beattie and S. Gugercin, Krylov-based minimization for optimal $\mathcal{H}_2$ model reduction, *Proceedings of the 46th IEEE CDC* (2007).

5. C.A. Beattie and S. Gugercin, Interpolatory Projection Methods for Structure-preserving Model Reduction, *Systems and Control Letters*, submitted.

6. L.C. Berselli, T. Iliescu and W.J. Layton, *Mathematics of Large Eddy Simulation*, Springer Verlag (2005).

7. V.P. Bongolan-Walsh, J. Duan, P. Fischer, T. Iliescu and T. Ozgokmen, Impact of Boundary Conditions on Entrainment and Transport in Gravity Currents, *Applied Mathematical Modelling*, Vol 31 (2007).

8. J. Borggaard, Optimal Reduced-Order Modeling for Nonlinear Distributed Parameter Systems, in *Proceedings of the 2006 American Control Conference*, Paper WeB13.3 (2006).

9. J. Borggaard, A. Hay and D. Pelletier, Interval-Based Reduced-Order Models for Unsteady Fluid Flow, *International Journal of Numerical Analysis and Modeling*, Vol 4 (2007).

10. J. Borggaard and T. Iliescu, Approximate Deconvolution Boundary Conditions for Large Eddy Simulation, *Applied Math Letters*, Vol. 19, pages 735–740 (2006).

11. J. Borggaard and T. Iliescu, A numerical investigation of the boundary commutation error in large eddy simulation, in the Proceedings of ECCOMAS CFD 2006, June (2006).

12. J. Borggaard, T. Iliescu, H. Lee, J.P. Roop and H. Son, A Two-Level Smagorinsky Model, *Multiscale Modeling and Simulation*, accepted.

13. J. Borggaard, T. Iliescu and J.P. Roop, A Bounded Artificial Viscosity Large Eddy Simulation Model, *SIAM Journal on Numerical Analysis*, in press.

14. J. Borggaard, T. Iliescu and J.P. Roop, An Improved Penalty Method for Power-Law Stokes Problems, *Journal of Computational and Applied Math*, accepted.

15. J. Borggaard, M. Stoyanov and L. Zietsman, A Penalty Method Approach to LQR Control for Saddle Point Problems in *Proceedings of the International Conference on Nonlinear Problems in Aviation and Aerospace 2006*, Chapter 64 (2006).

16. E. Colin, S. Étienne, D. Pelletier and J. Borggaard, A General Sensitivity Equation Formulation for Turbulent Heat Transfer, *Numerical Heat Transfer: Part B, Fundamentals*, Vol. 49, No. 2, pages 125–153 (2006).

17. J. Cori, S. Étienne, A. Hay, D. Pelletier, J. Trépanier and J. Borggaard, Optimal Design of Airfoils using NURBS and a Continuous Sensitivity Equation Method, in *Proceedings of the 45th AIAA Aerospace Sciences Meeting and Exhibit*, AIAA Paper 2007-1129 (2007).

18. R. Duvigneau and D. Pelletier and J. Borggaard, An Improved Continuous Sensitivity Equation Method for Optimal Shape Design in Mixed Convection, *Numerical Heat Transfer: Part B, Fundamentals*, Vol. 50, No. 1, pages 1–24 (2006).

19. R. Duvigneau, D. Pelletier and J. Borggaard, Optimal Shape Design in Mixed Convection using a Continuous Sensitivity Equation Approach, in *Proceedings of the 38th AIAA Thermophysics Conference*, AIAA Paper Number 2005-4823 (2005).

20. P. Edmond, D. Pelletier, S. Etienne and J. Borggaard, A Sensitivity Equation Method for Compressible Subsonic Laminar Airfoil Flows, in *Proceedings of the 23rd AIAA Applied Aerodynamics Conference*, AIAA Paper Number 2005-4601 (2005).

21. P. Edmond, D. Pelletier, S. Etienne, A. Hay and J. Borggaard, Application of a Sensitivity Equation Method to Compressible Subsonic Impinging Jets, in *Proceeding of the 44th AIAA Aerospace Sciences Meeting and Exhibit*, AIAA Paper Number 2006-0909 (2006).

22. S. Gugercin, An iterative SVD-Krylov method for model reduction of large-scale dynamical systems, *Linear Algebra and its Applications*, submitted.

23. S. Gugercin, An iterative SVD-Krylov based method for model reduction of large-scale dynamical systems, *Proceedings of the 44th IEEE CDC*, December 2005.

24. S. Gugercin and A.C. Antoulas, Model reduction of large scale systems by least squares, *Linear Algebra and its Applications*, Vol. 415, pp. 290-321, 2006.

25. S. Gugercin, A.C. Antoulas, and C.A. Beattie, Rational Krylov methods for optimal $\mathcal{H}_2$ model reduction, *SIAM Journal on Matrix Analysis and Applications*, accepted.

26. S. Gugercin, A. C. Antoulas and C. A. Beattie, A rational Krylov Iteration for optimal $\mathcal{H}_2$ model reduction, *Proceedings of the 17th International Symposium on Mathematical Theory of Networks and Systems*, Kyoto, Japan, July 2006.

27. S. Gugercin and K. Willcox, Krylov projection framework for Fourier model reduction, *Automatica*, 2007.

28. H. Hristova, S. Étienne, D. Pelletier and J. Borggaard, A Continuous Sensitivity Equation Method for Time-dependent Incompressible Laminar Flows, *International Journal for Numerical Methods in Fluids*, Vol. 50, No. 7, pages 871–844 (2006).

29. F. Ilinca, D. Pelletier and J. Borggaard, A Continuous Second Order Sensitivity Equation Method for Time-Dependent Incompressible Laminar Flows, in *Proceedings of the 17th AIAA Computational Fluid Dynamics Conference*, AIAA Paper Number 2005-5252 (2005).

30. T. Ozgokmen, T. Iliescu, P.F. Fischer, A. Srinivasan and J. Duan, Large Eddy Simulation of Stratified Mixing in Two-Dimensional Dam-Break Problem in a Rectangular Enclosed Domain, *Ocean Modelling*, Vol. 16, pages 106–140 (2007).

31. D. Pelletier, A. Hay, S. Etienne and J. Borggaard, The Sensitivity Equation Method in Fluid Mechanics, *European Journal of Computational Mechanics*, Vol. 17, pages 31–61 (2008).

32. E. Turgeon, D. Pelletier, J. Borggaard and S. Etienne, Application of a Sensitivity Equation Method to the $k - \epsilon$ Model of Turbulence, *Optimization and Engineering*, Vol. 8, pages 341–372 (2007).

Presentations at meetings, conferences or seminars

During this project, we have given more than fifty two presentations at meetings, conferences or seminars. This includes two tutorial talks (a short course and a plenary overview) on reduced-order modeling. In addition, graduate students attended regional meetings and gave presentations on their research involving reduced-order modeling for control problems.

C. Beattie

1. SIAM Annual Meeting, New Orleans, LA (July 2005).
2. Sandia National Lab, Albuquerque, NM (July 2005).
3. Ninth Copper Mountain Conference on Iterative Methods (April 2006).
4. **Plenary talk**: Sixth International Workshop on Accurate Solution of Eigenvalue Problems (IWASEP6), State College, PA (May 2006).
5. SIAM Annual Meeting, Boston, MA (July 2006).
6. GAMM-SIAM Conference on Applied Linear Algebra, Dusseldorf, Germany (July 2006).
7. Workshop on Structured Perturbations, and Distance Problems in Matrix Computations (March 2007).
8. 6th International Congress on Industrial and Applied Mathematics, Zurich, Switzerland (July 2007).

J. Borggaard

1. **Short Course**: Reduced-Order Model Development and Control Design (with K. Kunisch), SIAM Meeting on Control and its Applications, New Orleans, LA (July 2005).
2. SIAM Annual Meeting, New Orleans, LA (July 2005).
3. **Plenary talk**: Modeling and Optimization: Theory and Applications, Windsor, Ontario (July 2005).
4. AFOSR Computational Mathematics Program Review, Long Beach, CA (August 2005).
5. Workshop on Large-Scale Robust Optimization, Santa Fe, NM (August 2005).
6. Austrian Mathematical Society, Klagenfurt, Austria (September 2005).

7. Florida State University, Tallahassee, FL (November 2005).
8. Florida State University, School of Computational Science, Tallahassee, FL (January 2006).
9. 6th International Conference on Cooperative Control and Optimization, Gainsville, FL (February 2006).
10. 30th Annual Conference of the South African Society for Numerical and Applied Mathematics, Stellenbosch, South Africa (April 2006).
11. SIAM Annual Meeting, Boston, MA (July 2006).
12. 27th Annual Southeastern-Atlantic Regional Conference on Differential Equations, Greensboro, NC (October 2006).
13. 7th World Congress on Computational Mechanics, Los Angeles, CA (July 2006).
14. AFOSR Joint Program Review, Atlanta, GA (August 2006).
15. SIAM Conference on Computational Science and Engineering, Costa Mesa, CA (February 2007).
16. 14th International Conference on Finite Elements in Flow Problems, Santa Fe, NM (March 2007).
17. Computational and Mathematical Methods in Science and Engineering, Chicago, IL (June 2007).
18. SIAM Control Conference, San Francisco, CA (June 2007).
19. Air Force Research Laboratory, Wright-Patterson Air Force Base, OH (July 2007).
20. IFIP TC7 Conference on System Modelling and Optimization, Kraków, Poland (July 2007).
21. AFOSR Joint Program Review, Long Beach, CA (August 2007).
22. Model Order Reduction Seminar, MIT, Cambridge, MA (October 2007).
23. **Plenary Talk**: 28th Annual Southeastern-Atlantic Regional Conference on Differential Equations, Murray, KY (October 2007).
24. **Short Course**: Large Scale Optimization and Design, DoD High Performance Computing Program Office, University of Tennessee Space Institute, Arnold AFB, TN (February 2008).

S. Gugercin

1. SIAM Annual Meeting, New Orleans, LA (July 2005).

2. Bilkent University, Ankara, Turkey (August 2005).
3. Workshop on Large-scale Robust Optimization, Santa Fe, NM (August 2005).
4. Massachusetts Institute of Technology (MIT), Computational Prototyping Group, Cambridge, MA (October 2005).
5. Massachusetts Institute of Technology (MIT), Aerospace Computational Design Lab, Cambridge, MA (October 2005).
6. Ninth Copper Mountain Conference on Iterative Methods, Copper Mountain, CO (April 2006).
7. Conference on Adaptive Model Reduction Methods for PDE Constrained Optimization, Rice University, Houston, TX (May 2006).
8. Joint GAMM-SIAM Conference on Applied Linear Algebra, Duesseldorf, Germany (July 2006).
9. SIAM Conference on Computational Science and Engineering, Costa Mesa, CA (February 2007).
10. Workshop on Structured Perturbations, and Distance Problems in Matrix Computations, Bedlewo, Poland (March 2007).
11. Koc Univeristy, Turkey (July 2007).
12. Sixth International Congress on Industrial and Applied Mathematics, Zurich, Switzerland (July 2007).

T. Iliescu

1. The 58th Annual Meeting of the Division of Fluid Dynamics of the American Physical Society, Chicago (November 2005).
2. Workshop on Collaborations in the Mathematical Geosciences, poster presentation, Research Triangle Park, NC (October 2005).
3. Southeastern Section MAA and SIAM Southeast Atlantic Section Joint Meeting, Auburn, AL (March 2006).
4. University of Ottawa, Department of Mathematics and Statistics, Ottawa, Canada (May 2006).
5. Universita di Pisa, Department of Applied Mathematics, Pisa, Italy (May 2006).
6. Laboratory for Modeling and Scientific Computing MOX, Department of Mathematics "F. Brioschi", Politecnico di Milano, Milano, Italy (May 2006).
7. SIAM Conference on Computational Science and Engineering, Costa Mesa, CA (February 2007).

8. 6th International Congress on Industrial and Applied Mathematics, Zurich, Switzerland (July 2007).

9. **Plenary Talk**: Sandia CSRI Workshop on Mathematical Methods for Verification and Validation (August 2007).

10. AMS Spring Central Meeting, Bloomington, IN (April 2008).

Interactions and Transitions

Air Force Research Laboratory, Wright-Patterson Air Force Base, OH

Jeff Borggaard spent the summer at AFRL/VACA working with Chris Camphouse and James Myatt on a flow control problem. Efficient POD software using algorithms developed in this research and applicable to practical engineering reduced-order model-based control algorithms was provided to the AFRL researchers.

Daniel Sutton, Masters student in Mechanical Engineering, and partially supported on this project, spent a summer at the Air Vehicles Directorate working with Siva Banda, Chris Camphouse and James Myatt: He worked on computing low-order models based on Proper Orthogonal Decomposition. He studied the use of POD for coupled systems. The PI and Lizette Zietsman made a two day visit to the lab during Daniel's internship to discuss his research.

[Contacts: Chris Camphouse (937) 255-6326, James Myatt (937) 255-8498]

Synergistic Activities

1. Serkan Gugerin, with Karen Willcox from MIT, organized a two-part minisymposium on Model Reduction at the SIAM Annual Meeting, in Boston, MA, July 10-14, 2006.

2. Jeff Borggaard and Traian Iliescu co-organized *Emerging Finite Element Methods for Complex Flow* at the 2007 SIAM Conference on Computational Science and Engineering, Costa Mesa, CA (February 2007).

3. Jeff Borggaard co-organized *Model Reduction Methods for Flow in Porous Media* at the 2007 SIAM Conference on Mathematical and Computational Issues in the Geosciences, Santa Fe, NM (March 2007).

4. Joseph Ball and *Christopher A. Beattie* and *Serkan Gugercin* from Virginia Tech., Athanasios C. Antoulas from Rice University, and Tryphon T. Georgiou from University of Minnesota are the organizers of the 18th International Symposium on Mathematical Theory of Networks and Systems (MTNS 2008) in Blacksburg, VA, July 2008. There are several sessions on reduced-order modeling. Presentations by the co-PIs and graduate students on our reduced-order modeling research are scheduled.

Honors/Awards

Jeff Borggaard was awarded an ASEE Summer Faculty Fellowship and spent May-July 2007 at the AFRL Control Sciences Center of Excellence.

Serkan Gugercin was awarded an NSF Early Career Award in Computational and Applied Mathematics, 2007.